

Q4. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -1 & 3 \end{bmatrix}$

then compute $(A+B)$ and $(B-C)$. Also verify that $A + (B-C) = (A+B) - C$

Soln: $A+B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$

and $B-C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

$\therefore (A+B) - C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

and $A + (B-C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

Hence, $(A+B) - C = A + (B-C)$ is verified.

Q5. If $A = \begin{bmatrix} 2/3 & 1 & 5/3 \\ 1/3 & 2/3 & 4/3 \\ 7/3 & 2 & 2/3 \end{bmatrix}$ and $B = \begin{bmatrix} 2/5 & 3/5 & 1 \\ 1/5 & 2/5 & 4/5 \\ 7/5 & 6/5 & 2/5 \end{bmatrix}$

Compute $3A - 5B$.

Soln: $3A - 5B = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Q.6. Simplify $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

Soln: $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$
 $= \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\cos \theta \sin \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$
 $= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \sin \theta \cos \theta \\ -\cos \theta \sin \theta + \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
 $\therefore \sin^2 \theta + \cos^2 \theta = 1$

Q.7. Find X and Y if

(I) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

II $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 1 & -2 \\ -1 & 5 \end{bmatrix}$

Soln: (I) It is given that $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ — (I)

And $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Adding eqns I and II, we get

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} \text{ Ans}$$

Subtracting eqn II from eqn I, we get

$$2Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \text{ Ans}$$

Q.7 If it is given that

$$2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad \text{--- (I)}$$

$$\text{and } 3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \quad \text{--- (II)}$$

Multiplying eqn I by 2 and eqn II by 3 and then subtracting, we get

$$2(2x + 3y) - 3(3x + 2y) = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3 \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\Rightarrow 4x + 6y - 9x - 6y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$\Rightarrow -5x = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$\Rightarrow x = -\frac{1}{5} \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

$$\text{Then, from eqn (I), } 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2x$$

$$= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2 \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - \frac{4}{5} & 3 + \frac{24}{5} \\ 4 + \frac{22}{5} & 0 - 6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore y = \frac{1}{3} \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Ans.

Q.8. Find X , if $Y = \begin{bmatrix} 3 & 2 \\ 4 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Soln. It is given that:

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 3 & 2 \\ 4 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & 0-2 \\ 3-4 & 2-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Q.9. Find x and y , if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Soln. It is given that

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

By definition of equality

$$2+y = 5 \text{ and } 2x+2 = 8$$

$$\Rightarrow y = 5-2 \text{ and } 2x = 8-2$$

$$\Rightarrow y = 3 \text{ and } 2x = 6$$

$$\Rightarrow y = 3 \text{ and } x = 3$$

